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Reply to John Barker on Mathematics

John Barker's beautiful paper was full of elegant arguments against my view that mathematical proofs cannot really be beautiful or elegant, except in a metaphorical sense. The same is true of Barker's impressive paper if I am right! On first and second readings, I was almost ready to wave a white flag of surrender; his arguments, positive and negative, seemed unassailable. I contemplated a brief, if unsporting, "I agree" response. Nevertheless, I think I can offer something in reply.

In the last part of his paper (section 4), Barker gives a positive argument to the effect that ascriptions of "beauty" and "elegance" to mathematical proofs are similar to ordinary aesthetic judgements. They share the features of subjectivity and universality, for example. And there are no positive conditions for ascriptions of "elegance". Thus the cases of mathematical and ordinary "beauty" are parallel. I concede that these phenomena encourage the thesis that ascriptions of "beauty" and "elegance" to mathematical proofs are the same as ascriptions of "beauty" and "elegance" elsewhere.

My argument in the *Metaphysics of Beauty* (pp. 140–43) was that talk of beauty or elegance of proofs, like the beauty or elegance of theories or chess moves, machines and football goals is too closely tied to actually discharging a function to count as a genuine case of dependent beauty. In a case of dependent beauty, a thing, expresses, realizes or articulates, some function; but it need not actually discharge it. But the appropriateness of the attribution of "elegance" to a proof depends on its effectiveness in demonstrating some result. Hence ascriptions of "elegance" to proofs are a mere metaphor. So I argued.

The issue turns on the actual and possible separation of expressing a function from discharging it. This is where Barker makes his most impressive move against me, which is to point out that although the function of a proof is to derive a result, this can be done more or less elegantly. Inelegant proofs still demonstrate a result. So the elegance of a proof is not correlated with effectiveness, and therefore, may yet be an aesthetic property, since it is taken to be symptomatic of a dependent aesthetic property that it need not correlate with actual effectiveness (or effectiveness-conducive features).

Barker also raises explanatoriness as a feature of proofs. Explanatoriness is a virtue of proofs alongside soundness. But explanatoriness and elegance are also not correlated. So talking in terms of “elegance” cannot be a roundabout way of talking about explanatoriness.

I assumed a notion of dependent beauty according to which a thing that has dependent beauty can express a function while completely failing to fulfill it; and I argued that “elegance” applied to proofs is unlike this. Barker considers simply denying this, seeing the proof case as an exception to the general principle. But he is not content to rest with this argument. The general principle, incidentally, is not supposed to rest only on a few examples, but is supposed to derive from a quite general theoretical need for such a notion of dependent beauty. The only problem, I admit, is the lack of a proper detailed articulation of that central notion. This is something I am aware of, and hope others will take up as an intellectual project.

I appeared to be making an appearance/reality distinction when I argued that if elegance is a dependent aesthetic property, it can come apart from actually fulfilling a function. Barker’s move against my argument is to deny that an appearance/reality distinction is applicable in the case of proofs. In the case of mathematical proofs, there is no appearance of fulfilling the function (demonstrating a result) that can contrast with really doing so. In the mathematical case, Barker thinks that there is beauty in the fulfilling of function without the possibility of merely appearing to fulfill that function.

Barker is right that the correctness (soundness) and elegance of proofs can come apart; so a correct proof need not be elegant. But to what extent can an elegant proof not be correct? I do not think that my reasons for thinking that there are limits to this have to do with an appearance/reality distinction, but rather with what it is to be dependent beauty. That involves the admittedly under-specified but nonetheless important idea of the aesthetic expression (realization, articulation...) of a function, which is distinct from discharging it. The possibility of a beautiful or elegant functional thing, when it is not a well-function functional thing, is not in general an appearance/reality distinction.

It is true that the architectural case invites an appearance/reality analysis. A building may look aesthetically sturdy when it is not at all sturdy. But, firstly, consider an abstract sculpture that looks aesthetically delicate but which is sturdy. This delicacy is not a dependent aesthetic property. Secondly, consider the following case, which I think pulls apart three notions that we need to distinguish. (1) Many suspension bridges are elegant. This elegance is dependent elegance, let us assume (ignoring additional purely formal elegance). The bridge is elegant as a bridge, that is, as an expression of the bridge function of supporting people and transportation over a gulf. (2) The bridge may also appear flimsy and unable to perform that function. And lastly, (3) it is incredibly physically strong. Here the dependent elegance of the bridge has nothing to do with nonaesthetic perceptual appearances. Hence I do not think that Barker can counter that my argument is question-

begging against the mathematical case, where appearance and reality do not come apart.

Barker makes a lot of the possibility of inelegant but correct proofs, which I concede. But it is the other combination of elegant but incorrect proofs, that is important and that is doing the work in my argument. (Barker appears to recognize this before taking it to be a point about appearance and reality.) Can we remove the correctness of a proof leaving its elegance intact? It seems not. But this is puzzling and difficult to explain for the believer in aesthetic mathematical elegance. If the elegance of mathematical proofs is an aesthetic property of them, then why cannot there be elegant but ineffective proofs? The best explanation, surely, is that "elegance" does not denote an aesthetic property as it does normally, and it is being used metaphorically when applied to mathematical proofs.

So I persist in believing. But I must admit that my confidence in this thesis has been shaken by Barker's powerful and penetrating critique. But I am still inclined to think that my descriptions of his paper as "powerful", "penetrating", "beautiful" and "elegant" are all metaphors!